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An Inventory Control Model for Deteriorating Items with Probabilistic Demand Under Delay In Payment and Budget Constraint

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Abstract:
In today's dynamic business environment, effective inventory control is vital for operational efficiency and financial sustainability, especially for deteriorating products under demand uncertainty and capital constraints. This study develops an inventory control model for deteriorating items with probabilistic demand under delay in payment and budget constraint. In real operations, inventory decisions face challenges from demand variability, product degradation, financial limits, and supplier payment policies. The model minimizes total inventory cost per unit time—including ordering, storage, degradation, shortage, and financial holding costs—while accounting for interest earned during credit periods. It examines two scenarios: payment delays ending before or exceeding the replenishment cycle. Decision variables include optimal order quantity, safety stock, and reorder point, subject to budget constraints. Numerical experiments demonstrate that delays beyond the cycle eliminate financial holding costs, significantly lowering total expenses. Sensitivity analysis shows high responsiveness of total cost and order quantity to ordering costs, demand levels, and deterioration rate, with marginal influence from financial parameters. These findings affirm the model's realistic, robust support for managing deteriorating items amid demand uncertainty and financial constraints.

Keywords*: Deteriorating item; Delay in payment; Demand probabilistic; Budget constraint.*

INTRODUCTION

Inventory control is an integral part of various life contexts, both at the household level and in business activities. The differences lie in the intensity of use, the types of goods stored, and the systems used to manage them. Inventory management within a company is a crucial and essential task for both large and small companies. Effective inventory management is a key pillar of operational efficiency because it minimizes storage costs while ensuring smooth production flow (Barnes, 2018; Firoozi et al., 2024; Glock et al., 2019). Inappropriate inventory decisions can lead to increased holding costs, the risk of stockouts, and wasted resources. Therefore, inventory control should be viewed as a strategic part of business decision-making (Goldberg et al., 2021; Pramudito et al., 2024; Saha & Ray, 2019; Setiawan & Rizkya, 2020). Therefore, inventory control plays a crucial role in maintaining the efficiency and sustainability of business operations.

In practice, inventory control issues become increasingly complex due to increasing demand uncertainty and product characteristics that experience quality degradation over time. Empirical evidence from the global food retail industry shows that perishable product losses account for 10–15% of total inventory value annually, with demand variability contributing significantly to this waste (Aziz et al., 2025; Gupta et al., 2021; Li & Zobel, 2020). Similarly, in the pharmaceutical sector, studies indicate that expired medications represent 3–5% of total drug inventory costs, largely driven by uncertain demand patterns and strict regulatory expiration policies. Consumer demand in many sectors, particularly retail and distribution, tends to be uncertain and fluctuating due to changes in market preferences, seasonal factors, and the dynamics of the business environment. This situation becomes even more challenging when companies handle perishable or deteriorating products, where delays in sales or excess inventory not only increase holding costs but also result in losses due to decreased product value. Recent studies on perishable product supply chains confirm that perishability poses challenges at various decision levels (strategic, tactical, and operational), especially when demand is stochastic and uncertain (Aziz et al., 2025; Gupta et al., 2021; Li & Zobel, 2020). The primary objective of an optimal ordering policy for deteriorating goods is to ensure adequate supply of goods to customers and avoid shortages, thus ensuring optimal system or model design to minimize total losses or maximize net profits, thereby maintaining business continuity.

Harris in 1915 was the first to introduce the concept of a lot-size formula to determine the amount of inventory orders (Erlenkotter, 1989). This formula was then developed for the calculation of EOQ (Economic Order Quantity) by (Wilson, 1934). Ghare & Schrader (1963) were the first to derive a mathematical form of the EOQ inventory model for deteriorating goods by explaining its influence on demand patterns and deterioration rates simultaneously, based on the formulation put forward for the dynamics of inventory levels in one cycle following the following differential equation:

$$\frac{dI(t)}{dt} = -\theta(t)I(t) - D(t), 0 \le t \le T \tag{1}$$

Where the notations are $I(t)$is the inventory level at time t, $D(t)$is the demand level function, $\theta(t)$is the deterioration rate with $0 < \theta < 1$as the deterioration rate, and $T$is the cycle time of the inventory system. In their model, the assumption used is that the demand function and the deterioration rate are constant, but this assumption does not always match the type of goods in which demand patterns can change over time and depend on the quality of the inventory.

Furthermore, several time-dependent demand patterns, such as constant demand and demand that increases or decreases linearly or exponentially, have been considered in the development of inventory systems. A heuristic approach for inventory systems for deteriorating goods with a linear demand trend dependent on time was proposed by Dave and Patel (1981) and Silver (1979). However, in real situations, inventory shortages can occur in inventory systems. Deb and Chaudhuri (1987) extended Silver's (1979) heuristic model by introducing the concept of shortages into inventory systems. Ouyang et al. (2005) developed the concept of inventory for deteriorating goods with a constant deterioration rate, an exponentially decreasing demand pattern, and inventory shortages.

Recent studies have also developed time-dependent demand characteristics by considering deterioration and stockouts in inventory systems. Inventory models for deteriorating goods, where demand depends not only on time but also on inventory levels, incorporate backorder policies to address shortages (Lesmono et al., 2024). Furthermore, studies have formulated optimal inventory policies for goods with time-dependent demand and demonstrated the influence of the shape of the demand function on the structure of the optimal solution (San-José et al., 2024). Kaushik (2025) developed an inventory model for deteriorating goods that integrates blockchain technology to improve supply chain information transparency and analyzes its impact on unit holding costs across various operational scenarios. These references indicate that the development of modern inventory models is increasingly moving towards more realistic system representations by considering deterioration, diverse demand characteristics, and stockout handling mechanisms in decision-making. Recent studies also examine EOQ models for deteriorating goods with generalized time-dependent demand patterns in both deterministic and continuous frameworks, where demand is modeled as a cubic function of time without allowing for stockouts (Behera et al., 2025). The goal of these various deteriorating inventory models is to minimize system costs or maximize system profits.

In real market situations, suppliers encourage customers or retailers to purchase in bulk by offering various discounts based on quantity and payment terms. Suppliers typically grant customers a certain period of deferred payment to settle their payment obligations, referred to as permissible delay in payment. During this period, retailers or customers can resell the goods, earn interest, and settle payments to suppliers. After this period ends, customers or retailers are required to pay higher interest to suppliers.

Goyal (1985) developed an inventory model for deteriorating goods with the assumption of a constant demand function and a constant deterioration rate, which shows that the existence of a delay in payment period given by suppliers can affect ordering decisions and the total inventory cost structure through the interaction between operational costs and financial components. Furthermore, Aggarwal and Jaggi (1995) developed a mathematical model to determine the optimal cost with the condition of permissible payment delays. Next, Goyal and Giri (2001) provided a comprehensive review of various inventory models for deteriorating goods, covering various assumptions related to demand patterns, deterioration rates, shortage policies, and payment schemes, and identifying the direction of research development in the field of inventory control. With a deferred payment scheme, companies gain cash flow flexibility but also face the implications of interest costs and financial risks that need to be systematically considered in inventory decision-making. Other studies have shown that permissible delay in payment schemes have developed into a critical component of modern inventory systems, particularly for deteriorating goods, because they simultaneously affect ordering decisions, total cost structures, and financial risks due to interest and cash flows (Chen & Teng, 2014, 2015; Soni et al., 2010). Recent literature shows that permissible delay in payment policies for deteriorating goods inventories are increasingly being modeled to incorporate different interest structures and their impact on ordering decisions and total costs (Kaushik, 2023; Mallick et al., 2020).

In operational practice, inventory ordering decisions are often constrained by a company's limited capital, known as a budget constraint. This concept represents the maximum limit on available funds so that the total cost of purchasing and inventory-related costs cannot exceed the available budget. The concept of a budget constraint has its roots in microeconomic analysis and has been adopted in inventory system modeling to reflect realistic financial conditions (Lipsey, 2018; Yuxia Xiao, 2025). In the inventory context, ignoring budget constraints can result in mathematically optimal order sizes that are financially unfeasible, especially when purchasing costs dominate the total cost structure. Developing an inventory model incorporating budget constraints demonstrates that capital limitations can significantly alter the structure of the optimal solution. Several studies have shown that the presence of budget constraints tends to result in more conservative order lot sizes, especially in deteriorating inventory systems, where stockpiling not only ties up capital but also increases the risk of damage and depreciation of goods. Furthermore, Sadjadi and Karimi (2025) developed a multi-item EOQ model for deteriorating goods with budget constraints, time-dependent holding costs, and tiered quantity discounts, as part of an effort to enrich the formulation of capital-constrained inventory, but the model still assumes deterministic demand and does not consider demand uncertainty or delayed payment policies.

Based on a systematic classification of the inventory control literature for deteriorating products, a significant research gap remains between the dominant modeling approaches used and the need to represent real-world conditions. A recent comprehensive review showed that approximately 68% of studies still adopt variable deterministic demand and 16.5% use constant deterministic demand, while the more realistic probabilistic demand approach is only used by approximately 8% of studies (Karimi, 2025). In the deterioration dimension, approximately 50% of studies still focus on constant deterministic deterioration due to its analytical simplicity, although this approach does not fully reflect the dynamics of product quality in practice. Furthermore, only approximately 36.5% of studies explicitly include financial aspects such as permissible delay in payment, while the remaining 63.5% still ignore financial factors despite their proven significant influence on ordering decisions and inventory cost structure (Karimi, 2025). This condition indicates that most of the literature still relies on simplified and partial assumptions.

This gap becomes even more apparent when viewed from the perspective of system constraints and solution methods. A literature review reveals that approximately 78% of studies still ignore real inventory constraints, including budget constraints, and only about 22% explicitly incorporate these constraints into their model formulations (Karimi, 2025). Furthermore, despite increasing model complexity, the majority of studies still rely on heuristic or local solutions, while studies that successfully obtain an exact global optimal solution are still relatively limited. Based on these conditions, this study aims to bridge this literature gap by developing an inventory control model that simultaneously integrates probabilistic demand, constant deterministic deterioration, permissible delay in payment, and budget constraints. The model solution is analyzed using a global optimal solution approach and evaluated through sensitivity analysis to assess the effect of key parameter changes on solution behavior and inventory policy stability. Thus, this study is expected to not only contribute theoretically to the development of the literature but also produce inventory policies that are more realistic, applicable, and relevant for decision-making in real business environments.

Based on the identified research gaps, this study has three primary objectives: (1) to develop a comprehensive inventory control model that simultaneously integrates probabilistic demand, constant deterioration rate, permissible delay in payment, and budget constraints; (2) to derive the optimal solution using a global optimization approach and validate its mathematical properties; and (3) to conduct sensitivity analysis to evaluate the robustness of the model under varying parameter conditions. The benefits of this research are manifold. Theoretically, it contributes to advancing the inventory control literature by bridging the existing gap between simplified deterministic models and complex real-world conditions. Practically, the model provides decision-makers in retail, pharmaceutical, and food industries with a realistic and financially feasible framework for managing deteriorating inventories under demand uncertainty and capital limitations. Furthermore, the sensitivity analysis results offer strategic insights into which parameters require more careful management and monitoring, thereby supporting more informed and resilient inventory policies in dynamic business environments.

RESEARCH METHOD

This study used a quantitative analytical research design based on mathematical modeling to examine the problem of product inventory control for deteriorating items under conditions of demand uncertainty, permissible delay in payment, and budget constraint. The quantitative approach was chosen because the main objective of the research was to formulate and analyze a mathematical model that could produce optimal inventory policies through the optimization of the total cost of inventory. The entire research process focused on model development, numerical analysis, and solution evaluation.

The data used in this study were sourced from the company's operational records, which included historical data on demand, lead time, product deterioration rate, inventory cost structure, payment policy to suppliers, limited capital owned by the company, and other supporting data. The data were prepared through descriptive analysis and parameter estimation to ensure their consistency and suitability as inputs in the developed mathematical model.

The proposed model was a probabilistic inventory model with the objective function of minimizing the total cost of inventory. Total inventory costs were formulated as a combination of ordering costs, storage costs, inventory shortage costs, costs due to product deterioration, and financial costs arising from delayed payment policies and interest earned. Key decision variables in the model included optimal order size, safety stock, and reorder point. The model was formulated by taking into account the constraints of the level of service and the company's budget, so that the solution obtained was not only mathematically optimal but also operationally and financially feasible.

The model was solved through numerical analysis to obtain the optimal global solution of the formulated optimization problem. The analysis procedure included evaluating the feasibility of the solution against all model constraints as well as testing the optimal solution properties through the behavior of the total cost function. Furthermore, sensitivity analysis was performed by varying key parameters such as deterioration rate, demand variability, delay in payment period, and other relevant parameters to examine the effect of parameter changes on inventory decisions and total costs. This approach was used to assess the stability of the model and provide an analytical understanding of the implications of inventory policy under various scenarios of operational and financial conditions.

RESULTS AND DISCUSSION

1. Model Definition

Figure 1 is the inventory level in this research model that illustrates the dynamics of inventory level () in an inventory cycle with a time horizon, where the amount of inventory starts at the initial current value and then decreases gradually until it is close to zero at the end of the period. The pattern of decline that appears to be undulating reflects the existence of probabilistic demand, which is demand that changes randomly from one unit of time to the next so that inventory does not fall literally, but follows real demand fluctuations in the field.$I_0 T I_0 t = 0$

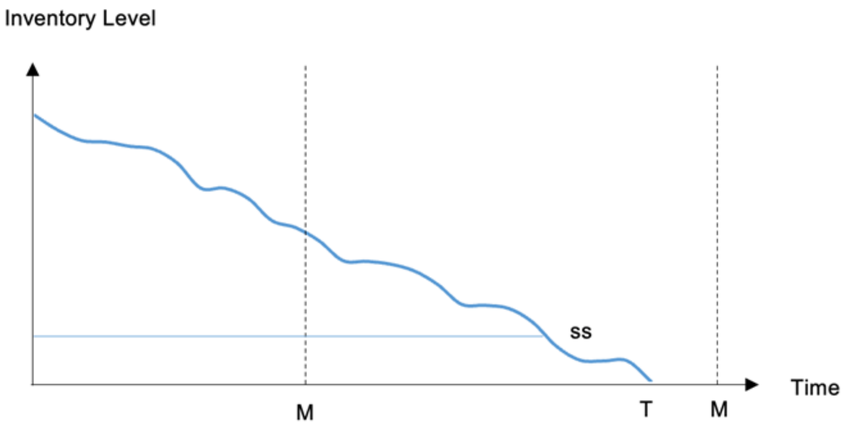

Figure 1 Research model inventory level

The vertical line shows the permissible M *delay in payment*. There are two cases in this study, namely:

1) The first case is when it is before, the company still has stock when the payment is due, so that capital costs or financial savings costs come into effect after . MTM
2) In the second case, when it is after, the inventory is exhausted before the payment obligation appears, so the company does not bear the financial burden of the goods that are still in storage.MT

Since demand is probabilistic, a safety reserve level (ss) is added to mitigate fluctuations in customer demand. Therefore, at the end of the inventory cycle, the inventory level is not heading to zero but close to the level ss**.** This graph also shows the interaction between stock consumption dynamics, demand uncertainty, and payment delay policies in the inventory model for deteriorating products.

2. Notation and Basic Assumptions

The developed model is formulated based on the following basic notation and assumptions.

Notation

The following mathematical notation is used in model development:

A : Cost of Ordering Per Order
B : the company's maximum budget/capital
$\bar{D}$: average requests per unit time
Q : Lot Size Booking
Q^* : Optimal booking lot size
$I(t)$: Inventory level at the time$t$

I_0 : the initial inventory level at the beginning of the cycle
I_{AVG} : average inventory level per cycle
θ : Rate of deterioration of goods
ss : safety stock
W : Physical storage cost in the warehouse per unit of goods per unit time
d : cost of damage per unit of goods
N : Expected amount of inventory shortage per cycle
s : standard deviation
S : Cost of Inventory Shortage Per Unit
F : financial savings cost per unit of goods per unit time
M : Payment delay deadline
e : Interest rate
p : purchase price per unit of goods
L : Lead Time Delivery
r : reorder point
T : Length of the order cycle
α : probability of inventory shortage
z_α : standard normal distribution standard value for levelα
$f(z_\alpha)$: Standard normal distribution density function
$\psi(z_\alpha)$: Partial Expectation of Standard Normal Distribution

Model Assumptions

The following are the assumptions used in the formulation of the research model:

1. The developed model applies to single items.
2. The characteristic of demand is probabilistic that follows a normal distribution pattern with average demand (and standard deviation .$\overline{D}$)(s)
3. The model developed is a simple probabilistic inventory model, where the size of the booking lot is fixed for each booking.(I_0)
4. The ordered goods come simultaneously with a constant planning time.(L)
5. An order is made when the goods in the storage area have reached the point of ordering.(r)
6. The price of goods sold is constant both in terms of the quantity of goods ordered and the time.(p)
7. The cost of storage in this research model is divided into cost of storage of space and financial cost of comparable value. The cost of storing value is proportional to the storage time of the item.(W)
8. The booking fee is constant for each time the booking is placed.(A)
9. The cost of inventory shortage is proportional to the quantity of goods or products that cannot be met.(S)
10. The bank's interest profit is proportional to the quantity of items sold.(e)
11. There are financial restrictions on the company.

12. Set-up time is not taken into account.
13. The level of service () or the possibility of inventory shortages is known and determined by the management.$\eta(\alpha)$
14. The supplier offers a way to pay for a number of items by deferring to a certain payment deadline.(M)
15. The number of damaged goods is proportional to the level of damage (goods to the amount of inventory in that one period.θ)
16. The amount of budget owned by the company is constant.
17. Assuming the replenishment comes instantly and there is no backorder, the initial inventory level at the beginning of the cycle is the same as the size of the order lot, so .$I_0 = Q$

3. Mathematical Formulation and Solution of The Model

The mathematical formulation of the model along with the step-by-step completion procedure is presented as follows. The first-order differential equation for inventory that takes into account the breakdown factor of goods (θ) and average customer demand is as follows:$(\overline{D})$

$$\frac{dI(t)}{dt} = -\overline{D}(t) - \theta I(t), 0 \le t \le T \tag{2}$$

The solution of the differential equation is as follows:

$$I(t) = \left(I_0 + \frac{\overline{D}}{\theta}\right) e^{-\theta t} - \frac{\overline{D}}{\theta} \tag{3}$$

To calculate the average inventory value per one period, then:

$$I_{AVG} = \frac{I_0}{ln\left(\frac{\theta I_0 + \overline{D}}{\overline{D}}\right)} - \frac{\overline{D}}{\theta} \tag{4}$$

Because the demand phenomenon is probabilistic, a number of safety reserves (ss) must be added to dampen fluctuations in customer demand, so that:

$$ss = z_\alpha s\sqrt{L} \tag{5}$$

$$I_{AVG} = \frac{I_0}{ln\left(\frac{\theta I_0 + \overline{D}}{\overline{D}}\right)} - \frac{\overline{D}}{\theta} + z_\alpha s\sqrt{L} \tag{6}$$

Ordering Cost per Unit Time $(\mathbf{C_0})$

The cost of ordering per unit of time in this study is defined as the cost of orders per order divided by the unit of time, by . Mathematically stated as follows:$(A)(T)T = {}^{I_0}/_{\overline{D}}$

$$C_0 = \frac{A}{T} = \frac{A\overline{D}}{I_0} \tag{7}$$

Warehouse Holding Cost per Unit Time $(\mathbf{C_w})$

Storage costs in warehouses per unit of time in this study are defined as storage costs per unit of goods per unit of time multiplied by the average physical inventory stored in the warehouse. So that the cost of storage per unit of time can be formulated as follows:(W)

$$C_W = W.I_{AVG} \tag{8}$$

$$C_W = W\left(\frac{I_0}{ln\left(\frac{\theta I_0+\overline{D}}{\overline{D}}\right)} - \frac{\overline{D}}{\theta} + z_\alpha s\sqrt{L}\right) \tag{9}$$

Deteriorating Cost per Unit Time $(\mathbf{C_d})$

Damage cost per unit time in this study is defined as the damage cost per unit of goods multiplied by the damage rate and the average of the physical inventory stored in the warehouse. So that the cost of damage per unit of time can be formulated as follows:(d) (θ)

$$C_d = d.\theta.I_{AVG} \tag{10}$$

$$C_d = d\theta\left(\frac{I_0}{ln\left(\frac{\theta I_0+\overline{D}}{\overline{D}}\right)} - \frac{\overline{D}}{\theta} + z_\alpha s\sqrt{L}\right) \tag{11}$$

Shortage Cost per Unit Time $(\mathbf{C_s})$

The magnitude of the cost of inventory shortage during the planning horizon is the multiplication between the expected amount of inventory shortage during the planning horizon and the unit cost of inventory shortage, then mathematically written as follows:$(C_s)(N_T)(S)$

$$C_s = \mathrm{S}.N_T \tag{12}$$

Where:

$$N_T = \frac{D}{I_0}N \tag{13}$$

$$N = s\sqrt{L}[f(z_\alpha) - z_\alpha\psi(z_\alpha)] \tag{14}$$

So that the cost of inventory shortage per unit of time in this study can be written as:

$$C_s = \frac{S.D}{I_0}\mathrm{x}s\sqrt{L}[f(z_\alpha) - z_\alpha\psi(z_\alpha)] \tag{15}$$

Financial Holding Cost per Unit Time $(\mathbf{C_F})$

In this section, it is divided into two cases as follows.

Case 1: $0 < M \leq T$. Financial costs per unit time are defined as:

$$C_F = \frac{F}{T}\int_M^T I(t)dt \tag{16}$$

Because, then:$I(t) = I(t) + ss$

$$\int_M^T I(t)dt = \int_M^T I(t)dt + ss(T-M) \tag{17}$$

Integral produces:$\int_M^T I(t)dt$

$$\int_M^T I(t)dt = \left(I_0 + \frac{\overline{D}}{\theta}\right)\frac{e^{-\theta M} - e^{-\theta T}}{\theta} - \frac{D}{\theta}(T-M) \tag{18}$$

$$C_F = \frac{F}{T}\left[\left(I_0 + \frac{\overline{D}}{\theta}\right)\frac{e^{-\theta M} - e^{-\theta T}}{\theta} - \frac{\overline{D}}{\theta}(T-M) + ss(T-M)\right], 0 < M \leq T \tag{19}$$

Case 2: $M \geq T$that is, when it is after, the inventory is exhausted before the payment obligation appears, so that the company does not bear the financial burden of the goods that are still stored.MT

$$C_F = 0, M \geq T \tag{20}$$

Interest Earned per Unit Time $(\mathbf{I_e})$

Case 1: $0 < M \leq T$

$$I_e = \frac{1}{T}\int_0^M (pD)e(M-t)dt = \frac{peDM^2}{2T} \tag{21}$$

Case 2:$M \geq T$

$$I_e = \frac{1}{T}\int_0^T (pD)e(M-t)dt = peD(M - \frac{T}{2}) \tag{22}$$

Inventory Total Cost

Inventory total cost is defined as the sum of the order costs C_0, warehouse storage costs, damage costs, financial savings costs, shortage costs minus the interest earned per cycle.$C_W C_d C_F C_S I_e$

$$TC = C_0 + C_W + C_d + C_s + C_F - I_e \tag{23}$$

Case 1: $\boldsymbol{0 < M \leq T}$

$$TC = \frac{A\bar{D}}{T} + W\left(\frac{I_0}{ln\left(\frac{\theta I_0 + \bar{D}}{\bar{D}}\right)} - \frac{\bar{D}}{\theta} + z_\alpha s\sqrt{L}\right) + d\theta\left(\frac{I_0}{ln\left(\frac{\theta I_0 + \bar{D}}{\bar{D}}\right)} - \frac{\bar{D}}{\theta} + z_\alpha s\sqrt{L}\right) +$$
$$\frac{S.\bar{D}}{I_0}\text{x}\, s\sqrt{L}[f(z_\alpha) - z_\alpha\psi(z_\alpha)] + \frac{F}{T}\left[\left(I_0 + \frac{\bar{D}}{\theta}\right)\frac{e^{-\theta M} - e^{-\theta T}}{\theta} - \frac{\bar{D}}{\theta}(T-M) + ss(T-M)\right] -$$
$$\frac{peDM^2}{2T} \tag{24}$$

Case 2: $\boldsymbol{M \geq T}$

$$TC = \frac{A\bar{D}}{T} + W\left(\frac{I_0}{ln\left(\frac{\theta I_0 + \bar{D}}{\bar{D}}\right)} - \frac{\bar{D}}{\theta} + z_\alpha s\sqrt{L}\right) + d\theta\left(\frac{I_0}{ln\left(\frac{\theta I_0 + \bar{D}}{\bar{D}}\right)} - \frac{\bar{D}}{\theta} + z_\alpha s\sqrt{L}\right) +$$
$$\frac{S.\bar{D}}{I_0}\text{x}\, s\sqrt{L}[f(z_\alpha) - z_\alpha\psi(z_\alpha)] + 0 - peD(M - \frac{T}{2}) \tag{25}$$

Budget Constraint

Because there are budget limitations, there are constraints, namely:

$$p.Q^* \leq B \tag{26}$$

Solution Search

The solution procedure used in this study follows an exact algorithm commonly used in the literature to determine the global optimum solution of an optimization problem by minimizing the total cost function and which is the decision variable. The steps of the settlement procedure are described as follows: $(TC)Q^*$

I. All the values of the available decision variables are searched by solving the equation: $\frac{dTC}{dI_0} = 0$
II. If for a variable value the result is obtained: $\frac{d^2TC}{dI_0{}^2} \geq 0$
and the total cost value meets the condition , then the optimum cost value is updated by setting .$TC(I_0) < TCTC = TC(I_0)$
III. Step II is repeated for all the value of the decision variables obtained in Step I.
IV. The global optimal solution is defined as the value of the decision variable that results in the minimum total cost value.Q^*TC
V. To check budget constraints, if then the solution is appropriate. $p.Q^* \leq BQ^*$

Numerical Examples

To illustrate the results of the research, the mathematical formula model will be applied to the following numerical values:

$\overline{D}$	: 228 unit per month
O	: 500.000 per order
θ	: 0,05
W	: Rp162 per unit per month
SS	: 22,5 unit per month
d	: 17.000 per unit
N	: 0,9309 1 unit≈
S	: Rp6000 per unit
F	: Rp243 per unit per month
M	: 1 month
e	: 0,05%
p	: IDR 17,000 per unit
η	: 95% ($z_\alpha = 1{,}65$)
s	: 52,78
L	: 2 day (0,067 month)
B	: IDR 10,000,000

By substituting the value in equation (5), the safety stock is obtained 22.5 to 23 units per month. And to obtain reorder points, it can be searched by using the equation, by substituting the equation then the value is obtained 37.68 38 units. The value of safety stock and reorder points obtained reflects the minimum inventory requirement that must be available to anticipate fluctuations in demand during the order waiting time. The establishment of a reorder point at this level ensures that the targeted level of service can be maintained while minimizing the risk of inventory shortages.$\approx r = \overline{D}L + ss \approx$

In Case 1, when the credit period is before the end of the inventory cycle, the company still has inventory at the time the payment obligation is due. This condition causes the incursion of financial savings costs for inventory that is still stored after time, thus increasing the total cost. This is reflected in Figure 2, where the curve is convex with a minimum point of Rp516,220 which is achieved at the optimal order size. The existence of financial costs after being the main factor that held back the reduction in total costs in this case.$M(M < T)MTCQ^* = 448M$

In contrast, in Case 2, when the credit period is after the end of the inventory cycle, the entire inventory has been exhausted before the payment obligation arises, so the company does not incur the cost of financial savings. The elimination of this financial cost component resulted in a significant decrease in total costs, as shown in Figure 3, with a minimum cost value of Rp501,830 in . A comparison of the two cases shows that Case 2 results in a lower total cost than Case 1, so it is economically and mathematically a more optimal solution.$M(M \geq T)Q^* = 483$

Furthermore, the feasibility of optimal solutions to budget limitations is tested using constraints. In Case 1, with the purchase price per unit, the optimal order size, and the budget, obtained. Because the value is smaller than the available budget, the optimal solution in Case 1 is declared to meet the budget constraint (feasible), with the remaining budget of Rp2,384,000. $pQ^* \leq Bp = 17{,}000Q^* = 448B = 10{,}000{,}000pQ^* = 7{,}616{,}000$As for Case 2, with the purchase price per unit, the optimal order size, and the budget, obtained. Because the value is smaller than the available budget, the optimal solution in Case 2 is declared to meet the budget constraint (feasible), with the remaining budget of IDR 1,789,000. $p = 17.000Q^* = 483B = Rp10.000.000pQ^* = 8.211.000$

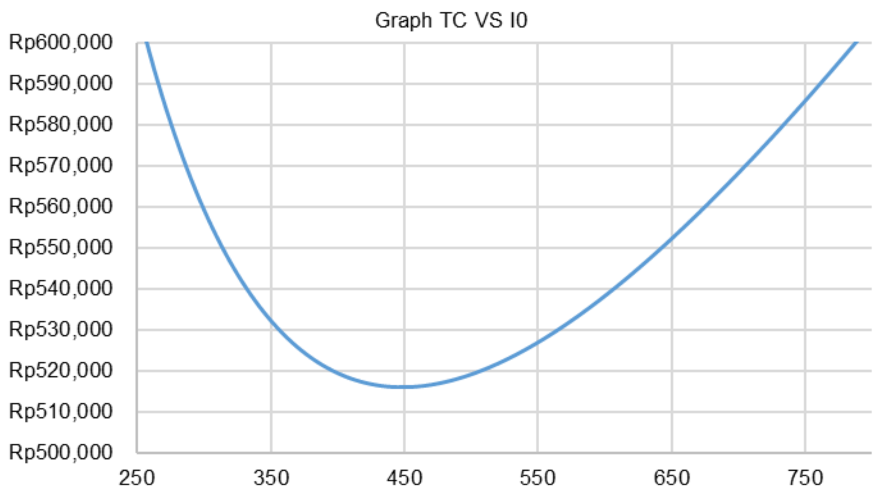

Figure 2 Concavity of total cost with respect to I0 (Case 1)

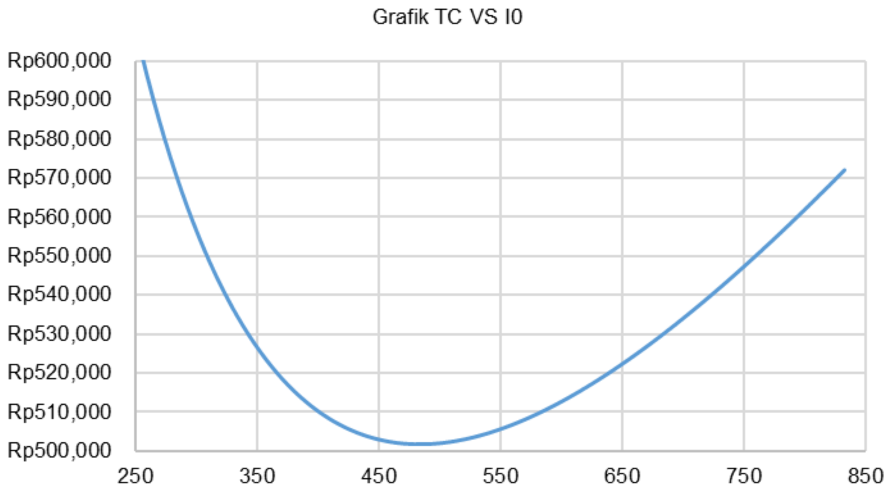

Figure 3 Concavity of total cost with respect to I0 (Case 2)

Sensitivity Analysis

The sensitivity analysis was then carried out using the parameters and results of the optimal solution in Case 2 as the basic condition, because this case resulted in a minimum total cost and met all model constraints.

Table 1. the parameters and results of the optimal solution in Case 2

Parameter	% Change in parameter	Value	TC	Q*
O	-75%	Rp125.000	Rp266.269	244
	-25%	Rp375.000	Rp438.603	418
	0%	Rp500.000	Rp501.830	483
	25%	Rp625.000	Rp557.504	541
	75%	Rp875.000	Rp653.999	641
D	-75%	57	Rp260.889	245
	-25%	171	Rp437.351	419
	0%	228	Rp501.830	483
	25%	285	Rp558.583	539
	75%	399	Rp656.954	637
θ	-75%	0,0125	Rp302.620	781
	-25%	0,0375	Rp444.606	542
	0%	0,05	Rp501.830	483
	25%	0,0625	Rp553.729	441
	75%	0,0875	Rp646.412	381
W	-75%	41	Rp469.459	515
	-25%	122	Rp491.333	493
	0%	162	Rp501.830	483
	25%	203	Rp512.395	474
	75%	284	Rp532.737	456
d	-75%	4250	Rp299.738	796
	-25%	12750	Rp443.631	544
	0%	17000	Rp501.830	483
	25%	21250	Rp554.701	439
	75%	29750	Rp649.333	378

F	-75%	61	Rp501.830	483
	-25%	182	Rp501.830	483
	0%	243	Rp501.830	483
	25%	304	Rp501.830	483
	75%	425	Rp501.830	483
e	-75%	0,000125	Rp501.740	485
	-25%	0,000375	Rp501.800	484
	0%	0,0005	Rp501.830	483
	25%	0,000625	Rp501.858	483
	75%	0,000875	Rp501.914	482
p	-75%	4250	Rp501.740	485
	-25%	12750	Rp501.800	484
	0%	17000	Rp501.830	483
	25%	21250	Rp501.858	483
	75%	29750	Rp501.914	482

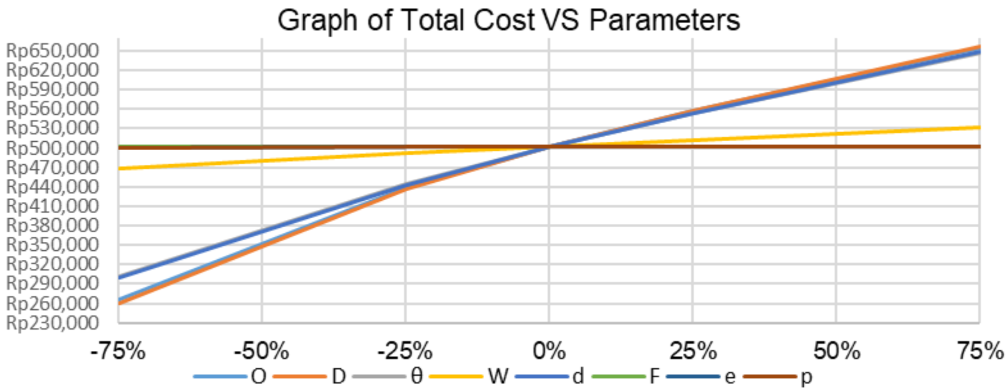

Figure 4 Graph of total cost versus parameters

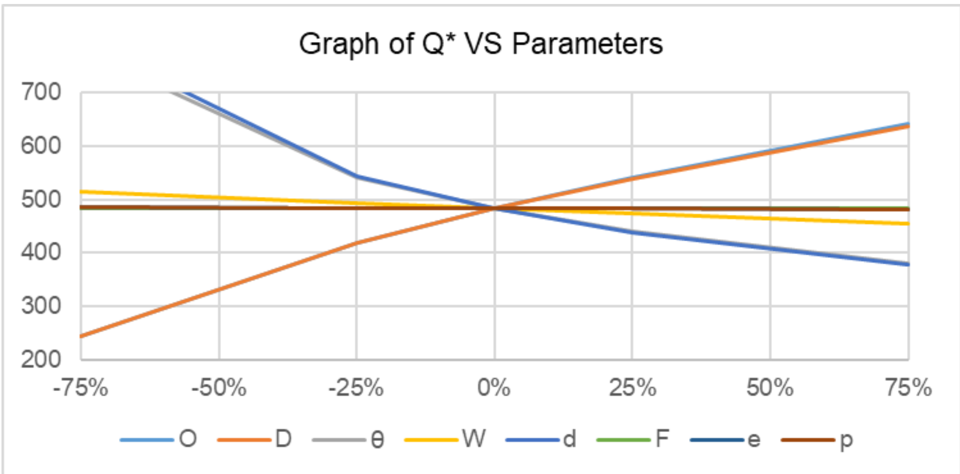

Figure 5. Graph of Q optimal versus parameters

The results of the sensitivity analysis show that the performance of the inventory model is strongly influenced by key operational parameters, in particular the cost of ordering (O) and demand (D). The ±75% change in both parameters resulted in significant variations in Total

Cost (TC), from Rp266,269 to Rp653,999 for O and from Rp260,889 to Rp656,954 for D, respectively. In addition, the optimal order size (Q*) increases monotonous as O and D increase, reflecting the classic trade-off between booking costs and storage costs, where increased booking costs or demand volumes encourage a larger lot booking policy to reduce order frequency. The parameters of deterioration rate (θ) and cost per unit of goods (d) also show a high level of sensitivity to TC, but with the opposite response direction Q*. Increases in θ and d consistently increased TC to more than 28% of base values, while Q* decreased sharply, from 781 to 381 and from 796 to 378 at a +75% change, respectively. This pattern indicates that an increased risk of damage or the economic value of the goods magnifies the inventory holding penalty, so the model optimally responds by reducing the lot size to reduce exposure to deterioration and capital costs.

In contrast, warehouse storage costs (W) show only a moderate effect on TC and Q*. A change in W of ±75% leads to a relatively small variation in TC (about ±6%), with a gradual decrease in Q* as W increases. This confirms that although storage costs are relevant, their role as cost drivers is under the influence of O, D, θ, and d in the model's cost structure. Meanwhile, the F parameter showed no influence on either TC or Q*, and the e and p parameters produced only very marginal changes. These findings indicate that within the range of parameters tested, optimal ordering policies are robust to variations in financial parameters, and inventory decisions are driven more by operational factors and physical risks of goods. Thus, these results confirm that the effectiveness of inventory control in the proposed model is highly dependent on demand management, order costs, and risk of deterioration mitigation, rather than on financial mechanisms alone. The results of the sensitivity analysis showed that demand uncertainty and the rate of deterioration had a more dominant influence on the total cost of inventory than financial parameters, thus confirming the importance of integrating operational aspects and physical risks of products in the design of inventory policy.

Managerial Implications

The results of this study provide several important managerial implications. First, inventory managers need to consider supplier payment policies as a strategic variable, not just a financial decision. Extending the delay in payment period to exceed the inventory cycle has been proven to be able to significantly reduce total costs by eliminating financial savings costs. Second, in inventory systems with deteriorating products and uncertain demand, the main focus of cost control should be directed to order cost management, demand estimation, and product damage risk mitigation, as these parameters have proven to be the main cost drivers. Third, the application of budget constraints in the model ensures that the resulting booking policy is not only mathematically optimal, but also financially feasible to implement. Thus, this model can be used as a practical decision-making tool for operational and financial managers in designing efficient, realistic, and sustainable inventory policies in the midst of demand uncertainty and capital constraints.

CONCLUSION

This research succeeded in developing an inventory control model for deteriorating products by considering probabilistic demand, permissible delay in payment policies, and budget constraints simultaneously. The proposed model formulates a function of total inventory costs per unit of time which includes order costs, warehouse storage costs, damage costs, inventory shortage costs, and financial components in the form of financial savings costs and interest earned during the credit period. By including budget constraints, the resulting solution is not only mathematically optimal but also financially feasible to implement. The results of the numerical analysis show that the optimal solution structure is greatly influenced by the position of the delay in payment period on the inventory cycle. When the credit period is after the end of the inventory cycle, the cost of financial savings can be completely eliminated, resulting in a lower total cost than the condition when the payment is due before the cycle ends. These findings confirm that supplier payment policies have a strategic role in determining the cost efficiency of inventory. Further sensitivity analysis revealed that order costs, demand levels, deterioration rates, and cost per unit of goods were the most dominant parameters influencing the total cost and optimal order size. In contrast, variations in financial parameters such as financial savings costs, interest rates, and purchase prices showed relatively marginal influences within the range of parameters tested. Overall, the results of this study show that the developed model is able to represent operational and financial conditions more realistically, and can be used as an effective decision-making framework in controlling the inventory of deteriorating products under uncertainty and capital limitations. This study still has limitations because it assumes a constant rate of deterioration and a single-product inventory system. Further research can develop models that take into account stochastic deterioration or multi-item systems under conditions of dynamic capital constraints.

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